

# Analytical solution for optimizing pollution load capacity in river segments

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**Abstract:** The problem of determining the pollution load capacity for a river section has an important meaning in protecting the water environment for the purpose of sustainable development. This is a complex optimization problem that only has an analytic solution in simple cases. This paper presents a method for obtaining an analytical solution for a river section in the case of dispersed waste sources distributed along the river, taking into account the influence of decay and dispersion processes. The results show a quantitative relationship between the decay coefficient, the dispersion coefficient and the load capacity of the river. The obtained results can be applied to more complex real-world problems.

## 1 Introduction

The determination of pollution load capacity in river systems plays a crucial role in water quality management and sustainable environmental development. As the pressures from urbanization, industrialization, and agricultural activities increase, rivers are becoming more vulnerable to pollution, necessitating effective methods to quantify and control pollutant discharge. Regulatory approaches such as the Total Maximum Daily Load (TMDL) [1,2] framework have been adopted in various countries to ensure that pollutant inputs do not exceed the assimilative capacity of water bodies, maintaining water quality standards for aquatic life and human use [3,4].

Analytical and mathematical models have long been used to simulate and optimize pollutant transport and transformation in riverine systems. These models account for critical processes such as advection, dispersion, and degradation [5,6]. However, many optimization problems regarding load capacity lack general analytical solutions and often require numerical approaches, especially when dealing with spatially distributed pollution sources [7,8]. Despite this complexity, analytical solutions—where obtainable—provide valuable insight and computational efficiency for water quality planning.

Previous studies have provided guidance and technical frameworks for implementing pollution control strategies, including Japan's Total Pollutant Load Control System (TPLCS), and comprehensive TMDL reports from states such as Maryland, USA [9-11]. These efforts highlight the importance of incorporating hydrodynamic and biochemical processes into load estimation and management decisions.

This paper presents an analytical solution for optimizing the pollution load capacity of a river segment with distributed waste sources. The model incorporates the effects of pollutant decay and dispersion, enabling a more accurate estimation of load capacities under constrained environmental conditions. By deriving closed-form solutions under specific assumptions, the study contributes a novel theoretical framework that can support more effective and efficient pollutant management strategies in

river systems. The findings also provide a foundation for extending these methods to more complex scenarios and integrating them into practical water quality control programs.

## 2 Materials and methods

### 2.1 Problem

In order to solve the problem analytically, we selected a 1D model with a river segment of length  $L$ , cross-section  $A$ , and a constant discharge  $Q$ . Concentration of influent waste in the river section is  $c_0$ , the reduction coefficient is  $\lambda$  (including decomposition and deposition). Discharge load distribution on both sides of the river is  $\rho(x)$ . Find  $p(x)$  such that the total load on both sides of the river into the river is the maximum. The constrain conditions are as follows:

- (1) Concentration of pollutants in the river section does not exceed the allowable standard  $C_{\max}$
- (2) Density of discharge load in the river does not exceed the limit  $\rho_{\max}$

### 2.2 Mathematical model

The river segment is shown on the  $x$  coordinate axis from coordinates  $a$ ,  $b$  (section  $L = [a, b]$ ). In the case of constant flow rate  $Q$ , the problem considered is stationary.

The distribution of pollutant concentrations is determined by the following differential equation:

$$v \frac{dc}{dx} = \alpha \frac{d^2c}{dx^2} - \lambda c + \rho(x) \quad (1)$$

Where  $c(x)$  [mg/L] pollutant concentration;  $v=Q/A$  [m/s] average flow velocity;  $\alpha$  [m<sup>2</sup>/s] eddy diffusion coefficient;  $\lambda$  [1/s] the degradation coefficient of the pollutant;  $\rho(x)$  [mg/L.s] discharge rate density along the river.

Constrain conditions:

- (1)  $0 < c(x) < C_{\max}$  ;  $x \in [a, b]$
- (2)  $0 < \rho(x) < \rho_{\max}$  ;  $x \in [a, b]$

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Boundary conditions (given the input concentration of the river section):

(1)  $c(a)=c_0$  (taking into account the concentration of pollutants from upstream)

Determining the total load of the river section is objective function:

Find  $\rho(x)$  with  $x \in [a, b]$  to  $TL = \int_a^b \rho(x) dx$  max.

Then the load capacity (LC) of the river section (including upstream load) is

$$LC = \int_a^b \rho(x) dx + c_0 Q \quad (2)$$

also reached its maximum.

For the convenience of finding analytical solutions, the above equations are reduced to dimensionless form, with dimensionless variables as follows:

$$\tilde{x} = \frac{(x-a)}{L}; \quad \tilde{c} = \frac{(c-c_0)}{(c_{max}-c_0)}; \quad \tilde{\rho}(\tilde{x}) = \rho(x) \quad (3)$$

Equation 2 (2) is rewritten by substituting Equation 3 (3) into equation 2:

$$vL \frac{d\tilde{c}}{d\tilde{x}} = \alpha \frac{d^2\tilde{c}}{d\tilde{x}^2} - \lambda L^2 \tilde{c} - \frac{\lambda L^2}{(c_{max}-c_0)} c_0 + \frac{L}{(c_{max}-c_0)} \tilde{\rho}$$

Divide both sides by  $vL$ , remove the "-" sign of the variables, we get:

$$\frac{dc}{dx} = A \frac{d^2c}{dx^2} - \Lambda c + R(x) \quad (4)$$

where

$$A = \frac{\alpha}{vL}; \quad \Lambda = \frac{\lambda L}{v}; \quad \tilde{c}_0 = \frac{c_0}{(c_{max}-c_0)};$$

$$R(x) = \frac{L}{v(c_{max}-c_0)} \rho(x) - \Lambda \tilde{c}_0$$

With constrain conditions:

$$-\Lambda \tilde{c}_0 = R_0 \leq R(x) \leq R_{max} = \frac{L\rho_{max}}{v(c_{max}-c_0)} - \Lambda \tilde{c}_0$$

$$-\tilde{c}_0 \leq c(x) \leq 1$$

With the boundary condition:

$$c(x=0)=0.$$

The objective function becomes:

$$TL = \int_a^b \rho(x) dx = v(c_{max}-c_0) \int_0^1 R(x) dx + L\lambda c_0$$

The objective function becomes finding maximum of  $R(x)$  where  $x$  belongs to the following interval  $[0, 1]$  for the integral (5):

$$\int_0^1 R(x) dx \quad (5)$$

With the conditions (6):

$$R_0 \leq R(x) \leq R_{max} \quad (6)$$

And with  $c(x)$  being the solution of the differential equation (7):

$$\frac{dc}{dx} = A \frac{d^2c}{dx^2} - \Lambda c + R(x) \quad (7)$$

Satisfy the boundary conditions (8):

$$c(x=0)=0 \quad (8)$$

then (9):

$$-\tilde{c}_0 \leq c(x) \leq 1 \quad (9)$$

## 3 Results

### 3.1 Results of determination of waste source distribution in case $A=0$

In the absence of the influence of dispersion processes (molecular diffusion and turbulent diffusion) corresponding to  $A=0$ , equation (7) is rewritten as:

$$\frac{dc}{dx} = -\Lambda c + R(x) \quad (10)$$

This equation with condition (5) has a unique solution:

$$c(x) = e^{-\Lambda x} \int_0^x R(s) e^{\Lambda s} ds$$

When  $R_{max}$  is very small,  $c(x)$  may not reach  $c_{max}=1$  in  $[0, 1]$ , in this case, in order to maximum the integral, we set  $R(x)=R_{max}$  over the whole interval  $[0, 1]$ . We have:

$$c(x) = e^{-\Lambda x} \int_0^x R(s) e^{\Lambda s} ds \leq R_{max}(1 - e^{-\Lambda x})/\Lambda$$

At  $x = 1$ , the right-hand side is only maximized and the value of  $c(x) = 1$ .

Therefore, the critical value  $R_c$  of  $R_{max}$  is determined by:

$$\frac{R_c(1 - e^{-\Lambda})}{\Lambda} = 1 \rightarrow R_c = \frac{\Lambda}{(1 - e^{-\Lambda})}$$

From here there is always the inequality  $R_c > \Lambda$ .

With  $R_{max} \leq R_c$

We have:  $R(x)=R_{max}$  with  $0 \leq x \leq 1$ .

and

$$c(x) = e^{-\Lambda x} \int_0^x R_{max} e^{\Lambda s} ds$$

or

$$c(x) = R_{max}(1 - e^{-\Lambda x})/\Lambda \quad (11)$$

so:

$$TL = \int_a^b \rho(x) dx = v(c_{max}-c_0) \int_0^1 R(x) dx + L\lambda c_0$$

Thus, the total load of the river will be:

$$TL = v(c_{max}-c_0)R_{max} + L\lambda c_0$$

The total load capacity of the river section will be:

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$$LC = v(c_{max} - c_0)R_{max} + L\lambda c_0 + c_0 Q$$

In this case  $R_{max} > R_c$ ,

We have:

$$\int_0^1 R(x) dx = \int_0^1 \left( \frac{dc}{dx} + \Lambda c \right) dx = c(1) + \Lambda \int_0^1 c(x) dx \quad (12)$$

So the solution where  $c(1)=1$  and the integral  $c(x)$  over  $[0,1]$  is maximized will be the optimal solution. This integral is maximal when  $c(x)$  soon reaches its maximum value. So the function  $R(x)$  needs to have the maximum value from the point  $x=0$ , that is,  $R=R_{max}$  in the interval  $[0, x_0]$  so that at  $x_0=1$ ,  $c(x)=1$  we have (13):

$$1 = \frac{R_{max}(1 - e^{-\Lambda x_0})}{\Lambda} \rightarrow x_0 = -\frac{1}{\Lambda} \ln \left( 1 - \frac{\Lambda}{R_{max}} \right) \quad (13)$$

On the segment  $[x_0, 1]$  to maximize the integral (10), we need to maintain  $c(x)=1$ , which means  $dc/dx=0$ . From equation 10 we have  $R=\Lambda$  because  $R > R_c > \Lambda$ .

So with  $R > R_c$  we have:

$$R = R_{max} \text{ with } 0 \leq x < x_0$$

$$R = \Lambda \text{ với } x_0 \leq x \leq 1.$$

So:

$$c(x) = R_{max} e^{-\Lambda x} (e^{\Lambda x} - 1) / \Lambda \quad 0 \leq x < x_0$$

$$c(x) = 1 \quad x_0 \leq x < 1$$

Thus:

$$\int_0^1 R(x) dx = R_{max} x_0 + \Lambda (1 - x_0) = \Lambda + \left( 1 - \frac{R_{max}}{\Lambda} \right) \ln \left( 1 - \frac{\Lambda}{R_{max}} \right)$$

$$TL = \int_a^b \rho(x) dx = v(c_{max} - c_0) \int_0^1 R(x) dx + L\lambda c_0$$

Thus, the total load will be:

$$TL = v(c_{max} - c_0) \left[ \Lambda + \left( 1 - \frac{R_{max}}{\Lambda} \right) \ln \left( 1 - \frac{\Lambda}{R_{max}} \right) \right] + L\lambda c_0 \quad (14)$$

The total load capacity of the river section will be:

$$LC = v(c_{max} - c_0) \left[ \Lambda + \left( 1 - \frac{R_{max}}{\Lambda} \right) \ln \left( 1 - \frac{\Lambda}{R_{max}} \right) \right] + L\lambda c_0 + c_0 Q \quad (15)$$

## 3.2 Results of determination of waste source distribution in case $A \neq 0$

In the case, the influence of the dispersion process (molecular diffusion and turbulent diffusion) are considered in equation 4, we have  $A \neq 0$ .

Similar to the case  $A=0$ , there exists a value of  $R_c$  so that when  $R_{max} < R_c$ , the concentration value  $c(x)$  does not exceed the value 1. So:

a. With  $R_{max} < R_c$

We have

$$R = R_{max}$$

General solution of the equation:

$$\frac{dc}{dx} = A \frac{d^2c}{dx^2} - \Lambda c + R(x)$$

satisfy the boundary conditions:

$$c(x=0) = 0$$

With  $0 \leq x \leq 1$  have the form:

$$c(x) = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x} + R_{max} / \Lambda \quad (16)$$

Where:  $\lambda_{1,2} = \frac{1 \pm \sqrt{1 + 4A\Lambda}}{2A}$ , with  $\lambda_1 < 0 < \lambda_2$ ;  $C_1, C_2$  are the coefficient which are obtained from boundary conditions below:

- At  $x=0$ :  $c(x=0)=0$

- At  $x=1$ :  $c(x)$  and its derivative satisfy the continuity condition with  $d(x)$  is the solution of equation (7) with  $R=0$ , in the interval  $[1, +\infty)$

- At  $x=+\infty$ ,  $d(x)$  is limited, so  $d(x)$  has a general solution of

$$d(x) = D_1 e^{\lambda_1 x}$$

From the above boundary conditions, we have a linear system equations for the coefficients  $C_1, C_2$  and  $D_1$

$$\begin{cases} C_1 + C_2 = -\frac{R_{max}}{\Lambda} \\ e^{\lambda_1} C_1 + e^{\lambda_2} C_2 - e^{\lambda_1} D_1 = -\frac{R_{max}}{\Lambda} \\ \lambda_1 e^{\lambda_1} C_1 + \lambda_2 e^{\lambda_2} C_2 - \lambda_1 e^{\lambda_1} D_1 = 0 \end{cases} \quad (17)$$

Solving the equations (17), we have:

$$C_1 = \frac{R_{max}(\lambda_1 - \lambda_1 e^{\lambda_2} + \lambda_2 e^{\lambda_2})}{\Lambda(\lambda_1 e^{\lambda_2} - \lambda_2 e^{\lambda_2})} \quad (18)$$

$$C_2 = -\frac{R_{max} \lambda_1}{\Lambda(\lambda_1 e^{\lambda_2} - \lambda_2 e^{\lambda_2})} \quad (19)$$

$$C_3 = \frac{e^{-\lambda_1} R_{max} [\lambda_1 e^{\lambda_1} - \lambda_2 e^{\lambda_2} - (\lambda_1 - \lambda_2) e^{1/2A}]}{\Lambda(\lambda_1 e^{\lambda_2} - \lambda_2 e^{\lambda_2})} \quad (20)$$

Substitute the coefficients in (15), since  $c(x)$  increases in the interval  $(0,1)$  thus reaching a maximum at 1 and  $R_c$  is found from the condition  $c(1)=1$ . We have:

$$R_c = \frac{\Lambda}{(1 - e^{\lambda_1}) + \frac{\lambda_1(e^{\lambda_1} - \lambda_2 - 1)}{\lambda_1 - \lambda_2}} \quad (21)$$

b.  $R_{max} > R_c$ :

$$\int_0^1 R(x) dx = \int_0^1 \left( \frac{dc}{dx} + \Lambda c - A \frac{d^2c}{dx^2} \right) dx = \Lambda \int_0^1 c(x) dx + [c(1) - c(0)] - A \left[ \frac{dc}{dx}(1) - \frac{dc}{dx}(0) \right] \quad (22)$$

with  $c(0)=0$  and  $\frac{dc}{dx}(1) = \lambda_1 c(1)$  we have:

$$\int_0^1 R(x) dx = \Lambda \int_0^1 c(x) dx + c(1)(1 - \lambda_2 A) + A \frac{dc}{dx}(0) \quad (23)$$

The above integral reaches its maximum when there exists a condition for all three terms to simultaneously attain their maxima. The first term reaches its maximum when  $c(x)$  increases as rapidly as possible to 1 and maintains the value 1 until  $x=1$ . This condition is entirely consistent with the conditions for the second and third terms to also reach their maxima, with  $c(1) = 1$  (since  $\lambda_2 < 0$  we have  $1 - \lambda_2 A > 0$ ) and  $\frac{dc}{dx}$  attaining its maximum. The optimization problem is thus reduced to the following problem:

Determine  $x_0$  such that  $c(x)$  increases as rapidly as possible to 1 on the interval  $[0, x_0)$ , and then,  $c(x)$  reaches its maximum value  $c(x)=1$  (from  $x_0$  to 1). To make  $c(x)$  increase as rapidly as possible, we set  $R=R_{max}$ . For  $c(x)=\text{const}=1$ , we have  $\frac{dc}{dx} = 0$  and  $\frac{d^2c}{dx^2} = 0$ , then from

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equation 4 deduce  $R=\Lambda$ . The problem now is determining  $x_0$ .

In the interval  $[0, x_0]$  the solution has the form (since  $R(x)=\text{const}=R_{\max}$ ):

$$c(x) = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x} + R_{\max}/\Lambda \quad (24)$$

With the condition  $c(0)=0$ ;  $c(x_0)=1$  and  $dc/dx(x_0)=0$ , we obtain a system equations to find  $C_1, C_2$  and  $x_0$ :

$$\begin{cases} C_1 + C_2 + \frac{R_{\max}}{\Lambda} = 0 \\ C_1 e^{\lambda_1 x_0} + C_2 e^{\lambda_2 x_0} + \frac{R_{\max}}{\Lambda} = 1 \\ C_1 \lambda_1 e^{\lambda_1 x_0} + C_2 \lambda_2 e^{\lambda_2 x_0} = 0 \end{cases} \quad (25)$$

Solving the system equations with the condition  $x_0 \in [0, 1]$ , we get the values of the constants  $C_1, C_2$  and  $x_0$ .

The total load of the river will be:

$$TL = \int_a^b \rho(x) dx = v(c_{\max} - c_0) \int_0^1 R(x) dx + L \lambda c_0 \quad (26)$$

in which

$$\int_0^1 R(x) dx = x_0 R_{\max} + (1 - x_0) \Lambda$$

The load capacity of the river section including the load from upstream will be:

$$LC = v(c_{\max} - c_0)[x_0 R_{\max} + (1 - x_0) \Lambda] + L \lambda c_0 + c_0 Q \quad (27)$$

## 4 Applies to specific river sections

Consider the river section with the following characteristics (Figure 1):

- Length  $L=1000$  (m)
- Discharge  $Q=200$  m<sup>3</sup>/s
- Cross-section  $A_s=300$  m<sup>2</sup>
- Velocity  $v=Q/A_s=0.67$  (m/s)
- Pollutants  $c$  (eg BOD, COD): affected by degradation and dispersion
- Decomposition coefficient (decay, deposition...):  $\lambda=0.2$ /day
- Partition coefficient (molecular and turbulent diffusion):  $\alpha=1$  (m<sup>2</sup>/s)
- Concentration of  $C$  at upstream inlet:  $c_0=4$  mg/L
- Permissible concentration:  $c_{\max}=15$  mg/L
- Maximum allowable discharge density near the riverbank:  $\rho_{\max}=0.1$  mg/L.s (equivalent to 8.64 kg/m/day/m<sup>2</sup>)

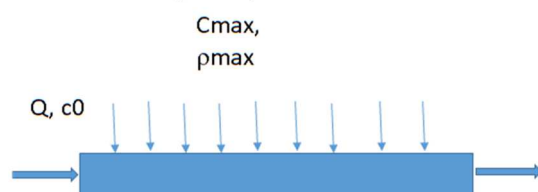


Figure 1 The illustration of a specific river section

In case of dispersion coefficient (molecular and turbulent diffusion):  $\alpha=0$  (m<sup>2</sup>/s)

Calculation results of load capacity and distance  $x_0$  according to the formulas (14),(15) and (13) are presented below (Table 1):

Table 1 The calculation results of load capacity and distance  $x_0$

Total Load Capacity (kg/day)	Load Capacity along river (kg/day)	Upstream (kg/day)	$x_0$ (m)
260076	190956	69120	73.35

The distribution of concentration  $c$  along the river and the load along the river is shown in Table 2, and Figures 2, 3 below:

Table 2 The distribution of concentration  $c$  along the river and the load along the river with  $\alpha=0$  (m<sup>2</sup>/s)

x (m)	c(x) (mg/l)	$\rho(x)$	
		kg/m/day/m <sup>2</sup>	kg/m/day
0	4	8.64	2592
10	5.50	8.64	2592
20	7.00	8.64	2592
30	8.50	8.64	2592
40	10.00	8.64	2592
50	11.50	8.64	2592
60	13.00	8.64	2592
73.3	14.99	8.64	2592
75	15	0.003	0.9
100	15	0.003	0.9
200	15	0.003	0.9
300	15	0.003	0.9
400	15	0.003	0.9
500	15	0.003	0.9
600	15	0.003	0.9
700	15	0.003	0.9
800	15	0.003	0.9
900	15	0.003	0.9
1000	15	0.003	0.9

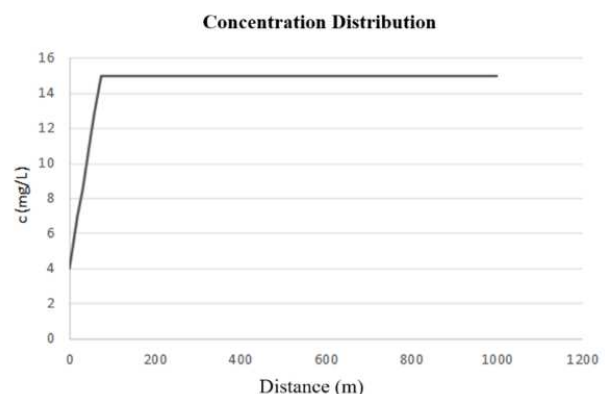


Figure 2 The distribution of concentration along the river

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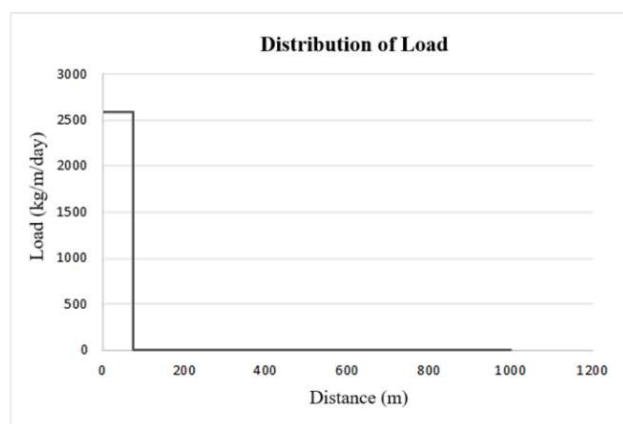


Figure 3 The distribution of load along the river

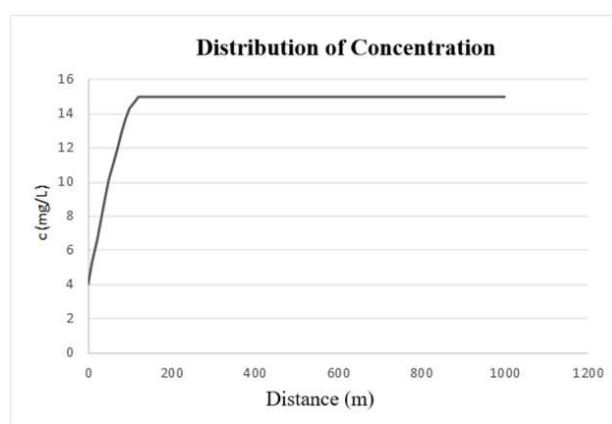


Figure 4 The distribution of concentration along the river

In case of dispersion coefficient (molecular and turbulent diffusion):  $\alpha = 40 \text{ (m}^2/\text{s)}$

Calculation results of load capacity and distance  $x_0$  according to the formulas 26, 27, 25 are presented below (Table 3):

Table 3 The calculation results of load capacity and distance  $x_0$

Total Load Capacity (kg/day)	Load Capacity along river (kg/day)	Upstream (kg/day)	$x_0$ (m)
388078	318958	69120	126

The distribution of concentration  $c$  along the river and the load along the river is presented in Table 4, and Figures 4, 5 below:

Distribution of concentration  $c$  along river and load along river with  $\alpha = 40$ .

Table 4 The distribution of concentration  $c$  along the river and the load along the river with  $\alpha = 40 \text{ (m}^2/\text{s)}$

x (m)	c(x) (mg/l)	$\rho(x)$	
		kg/m/day/m <sup>2</sup>	kg/m/day
0	4	8.64	2592
10.182	5.32	8.64	2592
20.365	6.61	8.64	2592
30.547	7.85	8.64	2592
40.730	9.04	8.64	2592
50.912	10.16	8.64	2592
61.095	11.21	8.64	2592
70.004	12.06	8.64	2592
80.187	12.93	8.64	2592
90.369	13.69	8.64	2592
100.552	14.29	8.64	2592
110.734	14.73	8.64	2592
120.916	14.97	8.64	2592
126.008	15	8.64	2592
127.000	15	0.003	0.9
600	15	0.003	0.9
700	15	0.003	0.9
800	15	0.003	0.9
900	15	0.003	0.9
1000	15	0.003	0.9

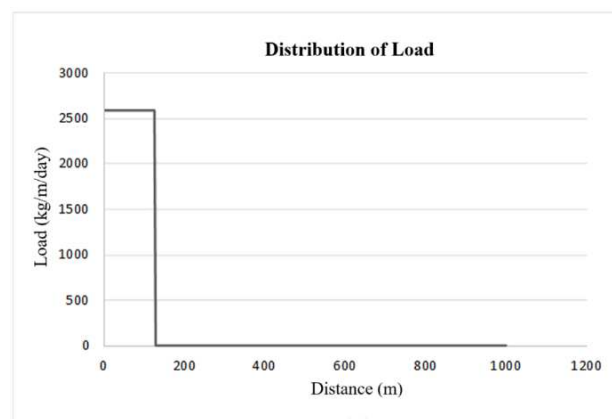


Figure 5 The distribution of load along the river

It can be seen that in the field  $\alpha = 40 \text{ (m}^2/\text{s)}$  the load carrying capacity of the river section increases to about 128002 kg/day (about 49%) compared to the case of  $\alpha = 0$ , and the distance  $x_0$  increases to about 50 m. Thus, the influence of dispersion coefficient is very large on the LC of the river section and should be considered in the calculation.

## Conclusion

This study successfully developed an analytical solution for optimizing the pollution load capacity of a river segment, addressing both theoretical and practical aspects of water quality management. The derived solutions for cases with and without dispersion effects provide a clear methodology for determining the maximum allowable pollutant discharge while adhering to environmental standards.

The application to a specific river segment highlighted the significant impact of dispersion coefficients on the river's load capacity, with a 49% increase observed when dispersion was considered. These findings underscore the importance of incorporating dispersion processes in pollution load calculations to achieve accurate and sustainable results.

Future research could extend this model to more complex river systems, including multi-dimensional flows



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and dynamic boundary conditions. Additionally, integrating real-time monitoring data could enhance the model's predictive capabilities. This work serves as a valuable tool for policymakers and environmental engineers in designing effective pollution control strategies, ultimately contributing to the preservation of water resources for future generations.

### References

- [1] JARRELL, W.M.: *Getting Started with TMDLs*. YSI Incorporated, Environmental Products Group, USEPA, Draft Guidance for Water Quality-Based Decisions, 1999.
- [2] United States Environmental Protection Agency, The TMDL Process, second ed. EPA 841-D-99-001, Office of Water (4305), Washington, DC, 20460, 1999.
- [3] United States Environmental Protection Agency (USEPA), Technical Guidance Manual for Developing Total Maximum Daily Loads, Book II: Streams and Rivers, Part I: Biochemical Oxygen Demand/Dissolved Oxygen and Nutrients/Eutrophication, EPA 823-B-97-002, Office of Water (4305), Washington, DC, 20460, 1997.
- [4] Official Journal of the European Community, 2000, Directive 2000/60/EC of the European Parliament and of the Council of 23 October establishing a framework for Community action in the field of water policy, OJ C L 327, 22.12.2000, 2000.
- [5] CLEMENTS, J.T., CREAGER, C.S., BEACH, A.R., BUTCHER, J.B., MARCUS, M.D., SCHUELER, T.R.: *Framework for a Watershed Management Program*. Project 93-IRM-4, Water Environment Research Foundation, Alexandria, 1996.
- [6] WANG, C., BI, J., AMBROSE, R.B.: Development and application of mathematical models to support total maximum daily load for the Taihu Lake's influent rivers, China, *Ecological Engineering*, Vol. 83, pp. 258-267, 2015.  
<https://doi.org/10.1016/j.ecoleng.2015.06.036>
- [7] YANG, L., MEI, K., LIU, X., WU, L., ZHANG, M., XU, J., WANG, F.: Spatial distribution and source apportionment of water pollution in different administrative zones of Wen-Rui-Tang (WRT) river watershed, China, *Environmental Science and Pollution Research*, Vol. 20, pp. 5341-5352, 2013.  
<https://doi.org/10.1007/s11356-013-1536-x>
- [8] ANH, D.T., BONNET, M.P., VACHAUD, G., MINH, C.V., PRIEUR, N., DUC, L.V., ANH, L.L.: Biochemical modeling of the Nhue River (Hanoi, Vietnam): Practical identifiability analysis and parameters estimation, *Ecological Modelling*, Vol. 193, No. 3-4, pp. 182-204, 2006.  
<https://doi.org/10.1016/j.ecolmodel.2005.08.029>
- [9] Ministry of the Environment, Japan, Guidance for Introducing the Total Pollutant Load Control System (TPLCS), 2011.
- [10] Maryland Department of the Environment, Total Maximum Daily Loads of Nitrogen, Phosphorus and Biochemical Oxygen Demand for the Lower Wicomico River Wicomico County and Somerset County, Maryland, 2001.
- [11] Maryland Department of the Environment, Total Maximum Daily Loads of Biochemical Oxygen Demand (BOD), Nitrogen and Phosphorus for Town Creek into which the Town of Oxford Wastewater Treatment Plant Discharges Talbot County, Maryland, 2002.

### Review process

Single-blind peer review process.